



**RESEARCH ARTICLE**

**MHD Flow through Porous Medium of Continuously Moving Vertical Surface with Heat Source**

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**ABSTRACT**

*In this section we study of a theoretical solution for MHD flow through porous medium of convection over a continuously moving vertical surface with heat source is obtained. A flow of this type represents a new class of boundary layer flow at a surface of finite length. The effects of various parameters like as permeability parameter ( $K$ ), Magnetic field ( $M$ ), Thermal Grashof number ( $Gr$ ), Solutal Grashof number ( $Gc$ ), Prandtl number ( $Pr$ ), Schimdt number ( $Sc$ ) and heat source ( $S$ ) on velocity, temperature and concentration profiles are discussed graphically. The expression for the skin-friction is obtained.*

**Key words:** Heat and mass transfer, MHD, permeability parameter, heat source

**INTRODUCTION**

Sakiadis (1961) studied the growth of the two dimensional velocity boundary layer over a continuously moving flat plate. Vajravelu (1968) studied the exact solutions for hydrodynamic boundary layer and heat transfer over a continuous moving, horizontal flat surface with uniform suction and internal heat generation/ absorption. Again, vajravelu (1988) extended the problem (1968) to a vertical surface.

The magneto hydrodynamics of electrically conducting fluids in the presence of magnetic field is encountered in many important problems in Geophysics and astrophysics. There has been a renewed interest in studying magnato-hydrodynamic (MHD) flow and heat transfer aspects in various geometries due to the effect of magnetic fields on the flow control and on the performance of many systems using electrically conducting fluids such as liquid metals, water mixed with little acid and others. Chakrabarti and Gupta (1979) considered hydro magnetic flow and heat and mass transfer over a stretching sheet. Vajravelu and Hadjinicolaou (1997) reported on convecting heat transfer in an electrically conducting fluid at stretching surface with uniform free stream. Other examples of studies dealing with hydromagnetic flows can be found in the papers by Gray (1979), Michiyoshi *et. al.* (1976), Fumizawa (1980), Muthucumaraswamy (2002) has been studied hydromagnetic flow and heat transfer on a continuously moving vertical surface. Recently, Reddy *et. al.* (2008) have studied on heat and mass transfer effects on MHD flow of continuously moving vertical surface with uniform heat and mass flux.

The purpose of this section is to report analytical solutions for the problem of heat and mass transfer by steady flow through porous medium of an electrically conducting in the presence of heat source and mass flux.

**FORMULATION OF THE PROBLEM**

Consider the steady, two-dimensional, laminar, incompressible flow of a viscous fluid on a continuously moving vertical surface in the presence of a uniform magnetic field, uniform heat and mass flux effect, issuing a slot and moving with uniform velocity  $u_w$  in a fluid at rest. Let the x-axis be taken along the direction of motion of the surface in the upward direction and y-axis is to be normal to the surface. The temperature and concentration levels near the surface are raised uniformly. The induced magnetic field, viscous dissipation is assumed to be neglected.

Now, under the usual Boussinesq's approximation, the flow field is governed by the following equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots(1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K'} u \quad \dots(2)$$

$$u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y^2} + S'(T' - T'_\infty) \quad \dots(3)$$

$$u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} \quad \dots(4)$$

The initial and boundary conditions are:

$$\left. \begin{aligned} u = u_w, \quad v = v_0 = \text{const.} < 0, \quad \frac{\partial T'}{\partial y} = -\frac{q}{k}, \quad \frac{\partial C'}{\partial y} = -\frac{j''}{k} \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad \dots(5)$$

We now introduce the following non-dimensional quantities:

$$Y = \frac{yv_0}{\nu}, \quad U = \frac{u}{u_w}, \quad T = \frac{T' - T'_\infty}{\left(\frac{qv}{kv_0}\right)}, \quad C = \frac{C' - C'_\infty}{\left(\frac{j''\nu}{kv_0}\right)}$$

$$\text{Pr} = \frac{\mu C_p}{k}, \quad \text{Sc} = \frac{\nu}{D}, \quad \text{Gr} = \frac{g\beta\nu\left(\frac{qv}{kv_0}\right)}{u_w v_0^2}, \quad \text{Gc} = \frac{g\beta^*\nu\left(\frac{j''\nu}{kv_0}\right)}{u_w v_0^2} \quad \dots(6)$$

$$M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \quad K = \frac{\nu}{K'}, \quad S' = \frac{v_0^2 S}{\nu}$$

In view of equation (6), the governing equation (2) - (4) reduce to the following non-dimensional form:

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} - \left(M + \frac{1}{K}\right)U = -GrT - GcC \quad \dots(7)$$

$$\frac{d^2T}{dY^2} + Pr \frac{dT}{dY} + SPrT = 0 \quad \dots(8)$$

$$\frac{d^2C}{dY^2} + Sc \frac{dC}{dY} = 0 \quad \dots(9)$$

The corresponding initial and boundary conditions in non-dimensional form are

$$\left. \begin{aligned} U = 1, \quad \frac{\partial T}{\partial Y} = -1, \quad \frac{\partial C}{\partial Y} = -1 \quad \text{at} \quad Y = 0 \\ U \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as} \quad Y \rightarrow \infty \end{aligned} \right\} \quad \dots(10)$$

Where  $Gr$ ,  $Gc$ ,  $M$ ,  $K$ ,  $Pr$  and  $Sc$  are the thermal Grashof number, Solutal Grashof number, Magnetic field parameter, permeability parameter, Prandtl number and Schmidt number respectively.

### SOLUTION OF THE PROBLEM

Solving equation (7) – (9) with boundary conditions (10), we get

$$U = (1 + A_2 + A_3)e^{-A_1Y} - A_2e^{-m_1Y} - A_3e^{-ScY} \quad \dots(11)$$

$$T = \frac{1}{m_1}e^{-m_1Y} \quad \dots(12)$$

$$C = \frac{1}{Sc}e^{-ScY} \quad \dots(13)$$

$$\text{Where } m_1 = \frac{Pr + \sqrt{Pr^2 - 4SPr}}{2}$$

$$A_1 = \frac{1}{2} \left\{ 1 + \sqrt{1 + 4 \left( M + \frac{1}{K} \right)} \right\}$$

$$A_2 = \frac{-Gr}{m_1 \left\{ m_1^2 - m_1 - \left( M + \frac{1}{K} \right) \right\}}$$

$$A_3 = \frac{-Gc}{Sc \left\{ Sc^2 - Sc - \left( M + \frac{1}{K} \right) \right\}}$$

The skin friction field is given as:

$$C_f = \frac{\tau'}{\rho u_w v_0} = - \left( \frac{dU}{dY} \right)_{Y=0}$$

$$= (1 + A_2 + A_3)A_1 - A_2m_1 - A_3Sc$$

## RESULTS AND DISCUSSION

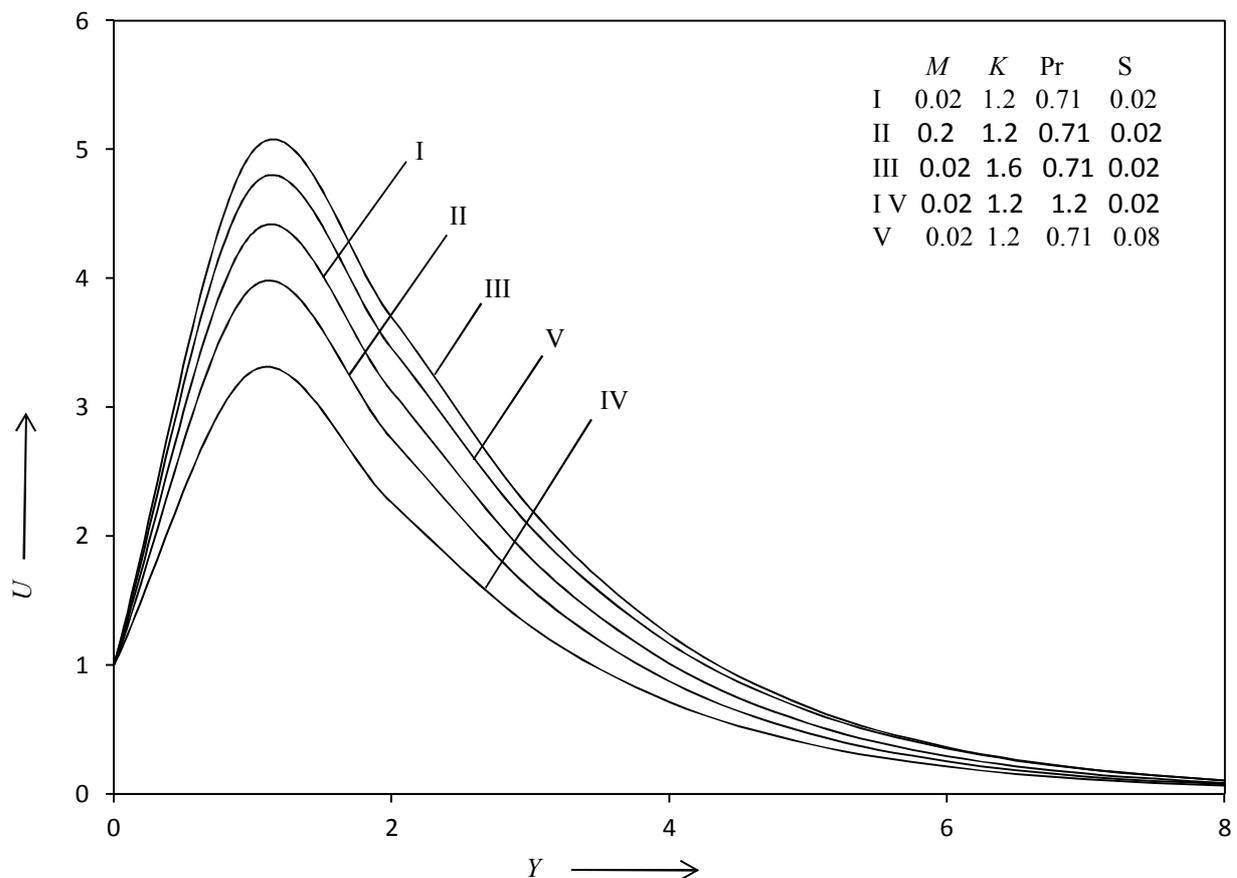
In the preceding sections, analytical solutions for the problem of heat and mass transfer by steady flow through porous medium of an electrically conducting in the presence of heat source and mass flux. The expression for the velocity, temperature and concentration were obtained.

Fig. 1, shows that the velocity profiles of boundary layer flow against span wise coordinate  $y$  for  $Gr = 4$ ,  $Gc = 5$ ,  $Sc = 0.6$  different values of permeability parameter ( $K$ ), Magnetic field ( $M$ ), Prandtl number ( $Pr$ ) and heat source ( $S$ ). It is obvious that the velocity decreases with increasing magnetic field parameter  $M$  and Prandtl number ( $Pr$ ). It is interesting to observe that for a relatively strong magnetic field, the distinctive peak in the velocity profile can be removed where the maximum velocity will be that of surface. It is observed that an increase in the value of  $K$  and heat source ( $S$ ), leads to increase in values of velocity.

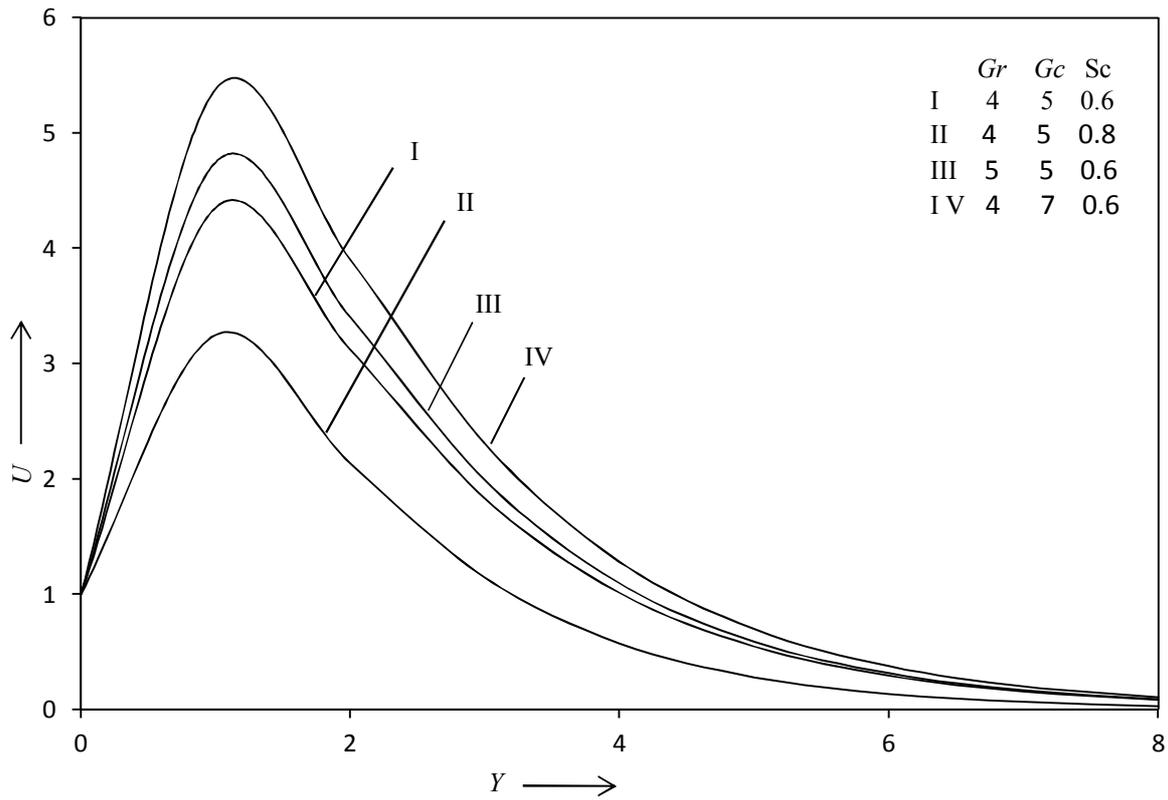
In Fig. 2, the velocity profiles of boundary layer flow against spanwise coordinate  $y$  for  $M = 0.02$ ,  $K = 1.2$ ,  $Pr = 0.71$  of different values of Thermal Grashof number ( $Gr$ ), Solutal Grashof number ( $Gc$ ) and Schmidt number ( $Sc$ ). It is observed that an increasing the value of  $Gr$  and  $Gc$ , leads to increase in values of velocity. But the velocity profile decreases with increasing the value of  $Sc$ .

The temperature profiles against span wise coordinate  $y$  are calculated forms eq. (12) and these are shown in Fig. 3. The effect of Prandtl number is very important for the temperature profiles. It is observed that the temperature decreases with increasing values of the Prandtl number, but heat source parameter ( $S$ ) increase leads to the temperature of fluid. It shows that the thermal-layer thickness decreases with increasing Prandtl number.

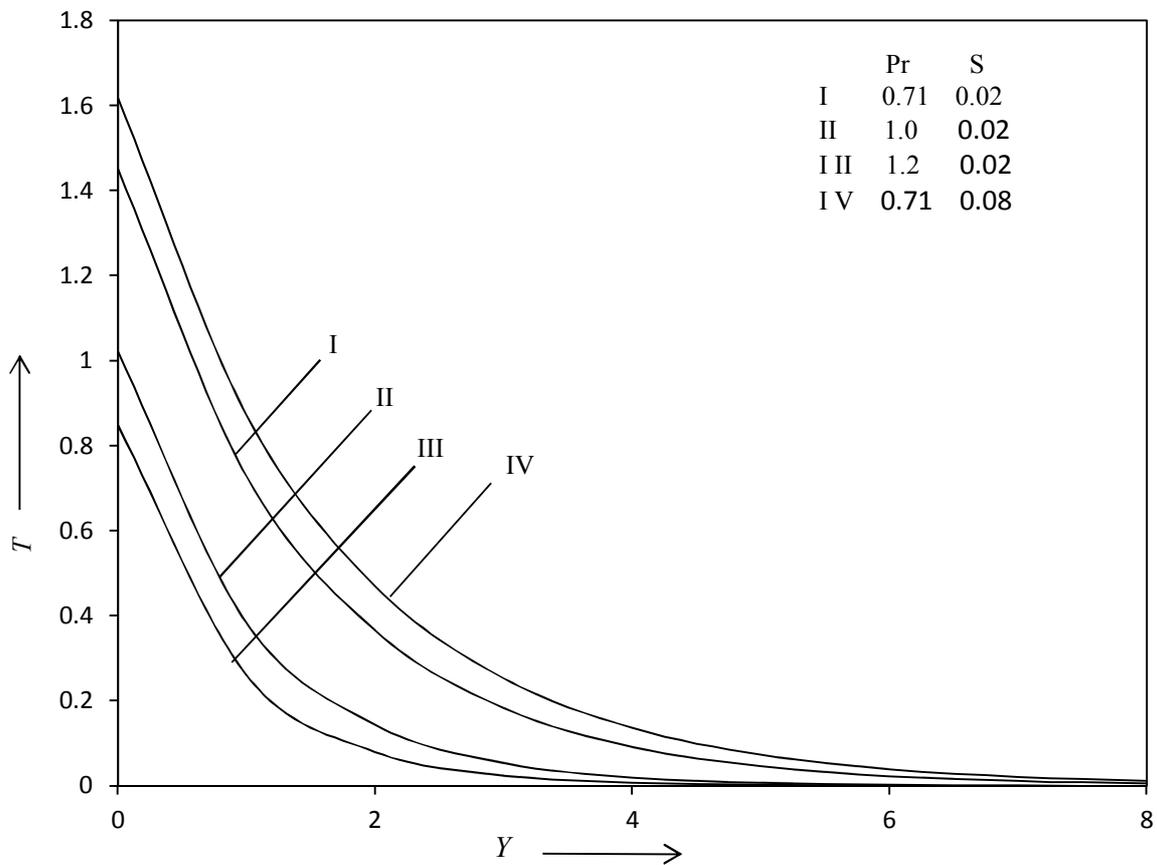
In Fig. 4, the effect of the concentration profiles against span wise coordinate  $y$  for the different Schmidt number  $Sc$ . The effect of Schmidt number is very important for the concentration profiles. It is observed that the concentration decreases with increasing values of the Schmidt number.



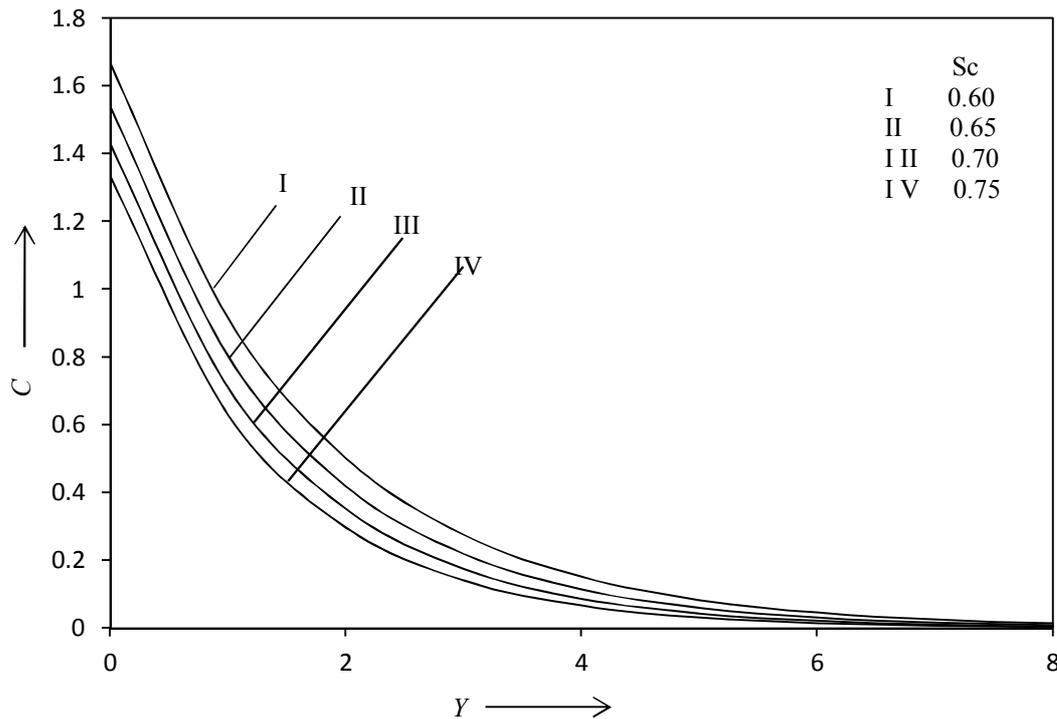
**Fig. 1:** The velocity profile for different value of  $M$ ,  $K$ ,  $Pr$  and  $S$ .



**Fig. 2:** The velocity profile for different value of  $Gr$ ,  $Gc$  and  $Sc$ .



**Fig. 3:** The temperature profile for different value of  $Pr$  and  $S$ .



**Fig. 4:** The concentration profile for different value of  $Sc$ .

## CONCLUSION

Analytical solutions of heat and mass transfer by steady flow of an electrically conducting in the presence of heat source  $x$  are obtained. The obtained results were compared with the previous works and were found to be in good agreement. The study concludes the following results.

1. It is obvious that the velocity decreases with increasing magnetic field parameter ( $M$ ) and Prandtl number ( $Pr$ ). It is observed that increasing the values of  $K$  and  $S$ , leads to increase in values of velocity.
2. It is observed that increasing the value of  $Gr$  and  $Gc$ , leads to increase in values of velocity. But the velocity profile decreases with increasing the value of  $Sc$ .
3. As the Prandtl number increases the temperature profile decreases. But the temperature increases with increasing the heat source parameter  $S$ .
4. It is observed that the concentration decreases with increasing values of the Schmidt number.

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